

Eliminating Thermal Violin Spikes from LIGO Noise

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(printed February 4, 2008)

Abstract

We have developed a scheme for reducing LIGO suspension thermal noise close to violin-mode resonances. The idea is to monitor directly the thermally-induced motion of a small portion of (a “point” on) each suspension fiber, thereby recording the random forces driving the test-mass motion close to each violin-mode frequency. One can then suppress the thermal noise by optimally subtracting the recorded fiber motions from the measured motion of the test mass, i.e., from the LIGO output. The proposed method is a modification of an analogous but more technically difficult scheme by Braginsky, Levin and Vyatchanin for reducing broad-band suspension thermal noise.

The efficiency of our method is limited by the sensitivity of the sensor used to monitor the fiber motion. If the sensor has no intrinsic noise (i.e. has unlimited sensitivity), then our method allows, in principle, a complete removal of violin spikes from the thermal-noise spectrum. We find that in LIGO-II interferometers, in order to suppress violin spikes below the shot-noise level, the intrinsic noise of the sensor must be less than $\sim 2 \times 10^{-13} \text{ cm}/\sqrt{\text{Hz}}$. This sensitivity is two orders of magnitude greater than that of currently available sensors.

04.80.Nn, 05.40.-a

I. INTRODUCTION

Suspension thermal noise is among the major sources of noise in the Laser Interferometer Gravitational Wave Observatory¹ (LIGO) [1]. In mature interferometric gravitational wave detectors, such as those of “LIGO-II” interferometers, it is predicted to be significant in the broad frequency band of 30-110Hz and to dominate other noise sources in the narrow peaks around frequencies of the standing wave modes (so-called violin modes) of the fibers on which the LIGO test masses are suspended. Fundamentally, suspension thermal noise arises from randomly fluctuating stresses of thermal origin in the suspension fibers. Currently, there are three general strategies for reducing the suspension thermal noise:

1. Reduce the mechanical losses in the test-mass suspension, thereby decoupling the suspension fiber’s motions from the suspension’s thermal reservoir and thence reducing the fluctuating stresses in the suspension fibers. There is a current vigorous experimental push in this direction (see, e.g., [2], [3], [4]).

2. Cool the suspension system [5], or some of its components². The above two strategies will not be discussed in this paper.

3. Use displacement sensors to monitor the motion of the suspension fibers, thereby recording independently the random forces responsible for the Brownian motion of the test mass (Langevin forces), and then optimally subtract this recorded force from the LIGO readout, thus achieving a partial compensation of the thermal noise. The general idea of thermal noise compensation has been widely discussed (See e.g.s Refs. [7] and [8]). A concrete version of it for broad-band suspension thermal noise in LIGO was conceived and analyzed by Braginsky, Levin, and Vyatchanin (BLV) [6]. Strigin and Vyatchanin have recently suggested a scheme which in principle allows analogous compensation for internal thermal noise [9].

A related approach to thermal noise compensation has been developed by Heidmann et. al. [10] and Pinard et. al. [11]. They have demonstrated — both experimentally and theoretically — a partial compensation of high-frequency internal thermal noise by implementing in hardware the subtraction of the Langevin force from the readout through a negative feedback loop. The result is an effective “cooling” of the mirror’s mechanical motions³.

In BLV, the broad-band 30-110 Hz component was targeted for reduction using a scheme

¹All results presented in this paper are equally applicable to LIGO’s international partners: VIRGO, GEO-600, TAMA, etc.

²A combination of cooling of the top suspension point, and adjusting location of the laser beam on the test mass to neutralize the thermal noise originating at the bottom of the suspension fiber, may significantly reduce suspension thermal noise in the broad frequency band [6].

³It seems to us that Heidmann et al’s [10] direct “cooling” technique requires a reference mirror which is much “quieter” than the mirror used in LIGO, so it is not clear how practical such an experimental arrangement is for LIGO. Nonetheless, [10] is an interesting experimental demonstration of the principle of dynamical thermal noise compensation.

where the motion of a suspension fiber is monitored along all its length, and an optimized average displacement is recorded and subtracted from the LIGO data. This scheme would be difficult to implement in practice, due to the fraction-of-a-micron proximity of the optical waveguide-based sensor and the fiber. In this paper, we instead propose to track the fiber's motion at a single point, which should be significantly easier to implement. This single-point-tracking does not allow significant reduction of the broad-band component of the suspension thermal noise, but when combined with optimal subtraction it should be quite effective in removing violin peaks from the thermal noise spectrum. The possibility of compensation of the thermal noise at violin resonances was briefly pointed out by Pinard et al. [11] in the context of the “cooling” technique.

Distinct from *reduction* of the suspension thermal noise, there are viable strategies to *filter out* narrow thermal violin-mode noise from the LIGO data [12]. By contrast with our scheme, a procedure of this kind would filter out *both* the noise and the signal around the violin spikes. This works well in most cases, since violin modes dominate other noise in very narrow frequency bands, and it is not likely that the signal in these particular bands would be significant. However, if the shot noise is reduced by increasing the laser power or by narrow-banding the interferometer's response, the broad bases of violin spikes could become dominant, and the genuine thermal noise compensation scheme, such as proposed here, could well be of use.

This paper is structured as follows: in Sec. II we give an intuitive introduction to the origin of suspension thermal noise and explain qualitatively why our method of thermal noise compensation should work.

In Sec. III we formally introduce a new readout variable, $x_{\text{readout}} = x_{\text{testmass}} + \alpha x_{\text{fiber}}$, where x_{testmass} and x_{fiber} are displacements of the test mass and of the independently monitored point of the fiber, and α is a frequency-dependent number which must be chosen in an optimal way. We then show how to use the Fluctuation-Dissipation theorem to compute the thermal noise in this new readout variable.

Section IV presents results of our calculations of the optimized parameter α and of the reduced thermal noise. We find (see Fig. 2) that on resonance the optimal value of α is $(1/\pi)m/M$, where m and M are the masses of the suspension fiber and the test mass, respectively.

In Sec. V we deal with the practical issue of the imperfection of the sensor which is used to monitor the suspension fiber. We find that if the sensor is infinitely precise (i.e., if it introduces no noise to the measurement), then the violin spike could be removed completely from the spectrum; however, in most realistic situations the effectiveness of our method will be severely limited by the sensor noise. For example, in order to bring down a violin spike to the shot-noise level at the violin frequency, one needs a sensor with the sensitivity $\sim 1.8 \times 10^{-13} \text{cm}/\sqrt{\text{Hz}}$ [cf. Eq. (31) of the text]; this sensitivity is significantly better than $\sim 10^{-11} \text{cm}$, which is the best sensitivity of currently used interferometric sensors of fiber motion [16]. Perhaps the situation will change when the next generation of displacement sensors comes to life.

II. MOTIVATION AND INTUITION

In this section we discuss intuitively the origin of suspension thermal noise and how to compensate it by monitoring devices.

Suspension thermal noise can be traced to randomly fluctuating stresses which are located in each suspension fiber. These stresses make the suspension fiber move in a random way; the fiber will make its test mass move randomly as well by pulling it sideways. It is this resulting random motion of the test mass that is referred to as the “suspension thermal noise”.

Clearly, the motion of the fiber and the test mass are not independent. The fiber’s random displacement is $\sim M/m$ greater than that of the test mass, where M and m are the masses of the test mass and the fiber respectively. BLV have argued that by monitoring the random motion of the fiber one gains information about the random forces acting on the test mass; one can then use this information to effectively reduce suspension thermal noise by orders of magnitude, at least in principle. Their scheme requires to measure x_{fiber} , which is a horizontal displacement approximately averaged over the fiber, then construct the new readout variable

$$x_{\text{readout}} = x_{\text{testmass}} + \alpha x_{\text{fiber}}, \quad (1)$$

where α is an optimization factor of order m/M that must be calculated theoretically. It is this new readout variable which has low thermal noise: The gravitational-wave signal is virtually unchanged by using this new readout variable, since α is very small (8×10^{-6} for LIGO).

The BLV scheme facilitates reduction of suspension thermal noise in the broad-band range of frequencies between the pendulum and the first violin peak, and if needed between the violin peaks. At the same time, their scheme has the disadvantage of requiring precise monitoring of the motion of the suspension fiber along all of its length⁴; at the moment there is, to our knowledge, no active experimental effort in this direction.

In this paper we propose a modification of the BLV scheme. The new scheme is easier to implement experimentally because it requires monitoring the fiber at a single point somewhere near the middle, rather than over all of its length. The requirement for the sensor sensitivity is also significantly less stringent than in the BLV proposal. However, single-point monitoring is not effective at removing the thermal noise in the broad-band region. Instead, our scheme is designed only to remove the thermal noise around the “violin spikes”, which pierce the photon shot-noise floor at frequencies above $\sim 300\text{Hz}$ for LIGO.

If we restrict our vision to a narrow band of frequencies close to some particular violin resonance, we will find that the motion of the fiber is remarkably simple: it looks like a standing wave with an integer number of half-wavelengths fitting the length of the fiber. The amplitude of the wave is a thermally fluctuating variable, but the shape does not fluctuate much. By sensing the fiber motion at a single point, we can monitor the fluctuating amplitude of the violin-resonance standing wave, and thus infer the random force with which the fiber pulls sideways on the test mass, at a frequency close to the violin resonance.

⁴Of course, one must monitor all of the suspension fibers.

As an example, consider the dynamics of a suspension fiber and its test mass close to the first violin resonance, which corresponds to a violin mode with half-wavelength equal to the length of the fiber. The fiber's horizontal displacement is given by

$$x_f(z) = A \sin(kz) \sin(\omega_v t), \quad (2)$$

where A is the (random) amplitude of the standing wave, z is distance from a point on the fiber to the top of the fiber, k is the wave vector, such that $kl = \pi$ (with l the length of the fiber), and $\omega_v = (\pi/l)\sqrt{Mgl/m}$ is the angular frequency of the first violin resonance. The force with which the fiber pulls sideways on the test mass is given by

$$F_f = -Mg \left(\frac{dx_f}{dz} \right)_{z=l} = Mg\pi \frac{A}{l} \sin(\omega_v t). \quad (3)$$

Assume for simplicity that the test mass has no tilt degree of freedom. This is actually the case for the currently planned 4-fibers suspension on each LIGO-II test mass. Then the laser beam spot measuring the test-mass position is displaced by the same amount as the test-mass center of mass:

$$x_{\text{testmass}} = -\frac{F_f}{M\omega_v^2} = -\frac{m}{\pi M} A \sin(\omega_v t). \quad (4)$$

We neglect the test mass radius compared to the wire length. Equation (4) relates the thermal motion of the fiber, which is almost completely determined by fluctuations in A , to the thermal motion of the test mass, at a frequency close to ω_v . If we choose to monitor the fiber at its midpoint, i.e., $x_{\text{fiber}} = x_f(z = l/2)$, then the linear combination

$$x_{\text{readout}} = x_{\text{testmass}} + \frac{m}{\pi M} x_{\text{fiber}} \quad (5)$$

is expected to have significantly reduced thermal noise at the first violin peak; cf. Eq. (1).

How much can the thermal noise be reduced when this new readout is introduced? What is the optimal readout variable for frequencies slightly off the violin resonance? In the next subsection we use the Fluctuation-Dissipation theorem to answer these questions.

III. THE NEW READOUT VARIABLE: ISSUES OF PRINCIPLE

For simplicity, let us assume that the test mass is hanging on a single fiber, and has no tilt degrees of freedom; see Fig. 1. In LIGO-II, the test mass is suspended on two fiber loops, which from the point of view of suspension thermal noise is equivalent to the test mass suspended on four independent fibers, and in this configuration the no-tilt assumption is realistic. The generalization of our scheme and analysis to a multi-fiber suspension is straightforward.

We assume that the gravitational-wave interferometer monitors the displacement of the test mass ⁵ x_{testmass} , and that an independent sensor S measures the displacement of the suspension fiber x_{fiber} at some point z_0 .

⁵Here it is assumed that the test mass moves as a rigid body, that is, we neglect the internal thermal noise due to fluctuations of the test-mass shape.

The observer should then record a new readout variable,

$$x_{\text{readout}} = x_{\text{testmass}} + \alpha x_{\text{fiber}}, \quad (6)$$

where α is a frequency-dependent coefficient which has to be chosen so that the thermal noise in x_{readout} is minimized.

The Fluctuation-Dissipation theorem [13] allows one to calculate the spectral density $S_x(f, \alpha)$ of the thermal noise in x_{readout} , where f is the frequency; see Refs. [14] and [6] for the general method of calculation, and see [15] for the first direct application of the Fluctuation-Dissipation theorem to computation of suspension thermal noise. First, we imagine that a generalized oscillating force $F(t) = F_0 \cos(\omega t)$ conjugate to x_{readout} is applied to the fiber + test mass system (here and elsewhere in this paper $\omega = 2\pi f$). We introduce such a force via the interaction Hamiltonian,

$$H_{\text{int}} = -F(t)x_{\text{readout}}. \quad (7)$$

As discussed in [6], and as is apparent from Eqs. (6) and (7), applying the generalized force F is equivalent to applying two simple “Newtonian” forces simultaneously: one, with a magnitude $F(t)$, to the test mass, and the other, with a magnitude $\alpha F(t)$, at a point $z = z_0$ on the fiber.

The next step is to calculate the motion of the test mass and the suspension fiber under the action of the generalized force, $F(t)$, and to work out the average power, W_{diss} , that would be dissipated as heat as a result of such motion. The thermal noise in x_{readout} is then given by

$$S_x(f, \alpha) = \frac{8k_B T}{\omega^2} \frac{W_{\text{diss}}}{F_0^2}, \quad (8)$$

where k_B is the Boltzmann constant, and T is the temperature of the suspension fiber (cf. Eq. (3.10) of [14]).

The last step is to choose α such that $S_x(f, \alpha)$ is as small as possible.

The oscillatory motion of the fiber+test mass under the action of the generalized force F is shown in Fig. 1. The fiber is strongly bent near three points⁶: at the bottom and top attachment points, and at the point on the fiber where we imagine applying αF (the location of the independent sensor). It is near these three points the regions in which the fiber bends and heat is produced by the fiber+test mass motion.

In order to compute the dissipated power W_{diss} , and then to use it in Eq. (8) to calculate the thermal noise, one must specify a model for dissipative losses in the suspension fiber. We assume that the time-averaged power dissipated as heat is given by

$$W_{\text{diss}} = f \left[\zeta_{\text{top}} \theta_{\text{top}}^2 + \zeta_{\text{bottom}} \theta_{\text{bottom}}^2 + \zeta_{\text{middle}} \theta_{\text{middle}}^2 \right], \quad (9)$$

⁶The length over which the fiber is bent is given by $\lambda = \sqrt{JE/Mg}$, where J is the geometric moment of inertia for the fiber crosssection (it equals to $\pi d^4/64$ for the circular crosssection of diameter d), and E is the Young modulus of the fiber material. For typical LIGO parameters λ is a small fraction of a centimeter, much less than the length of the suspension fiber

where θ_{top} , θ_{bottom} and θ_{middle} are the amplitudes of the oscillating top, bottom and middle angles respectively, see Fig. 1, and ζ_{top} , ζ_{bottom} and ζ_{middle} are frequency-dependent constants which are determined by the dissipation mechanism. If the source of dissipation is distributed homogeneously, then

$$\zeta_{\text{top}} = \zeta_{\text{bottom}} = 2\zeta_{\text{middle}} = \zeta. \quad (10)$$

The amplitudes of the oscillating angles θ_{top} , θ_{bottom} and θ_{middle} are worked out in Appendix A:

$$\theta_{\text{top}} = \frac{F_0}{M} \left[\frac{k [1 + \alpha \cos [k (l - z_0)]] - (\alpha/g) \omega^2 \sin [k (l - z_0)]}{gk \cos (kl) - \omega^2 \sin (kl)} \right], \quad (11)$$

$$\theta_{\text{bottom}} = \frac{F_0}{M} \left[\frac{k \cos (kl) + (\alpha/g) \omega^2 \sin (kz_0)}{kg \cos (kl) - \omega^2 \sin (kl)} \right], \quad (12)$$

and

$$\theta_{\text{middle}} = \frac{\alpha F_0}{Mg}. \quad (13)$$

Here M is the mass of the mirror, m is the mass of the suspension fiber, l is the fiber length, z_0 is the distance along the fiber from the top attachment point to the sensor,

$$k = \omega / \sqrt{Mgl/m}, \quad (14)$$

is the wavenumber of the standing wave induced in the fiber, and z_0 is the distance from the suspension top to the sensor.

The expressions for θ_{top} and θ_{bottom} do not take into account damping, and hence they diverge when ω approaches the violin-resonance angular frequency. We correct for this by replacing ω and k by $\omega(1 + i/Q_n)$ and $k(1 + i/Q_n)$ when ω is close to the n -th violin-resonance⁷, and by taking the absolute value of the now complex expressions for θ_{top} and θ_{bottom} . Here Q_n is the quality factor of the n -th violin mode; it can be shown, using Eqs. (9) and (10), that

$$Q_n = \frac{\pi Mgl}{2\zeta(f_n)}, \quad (15)$$

for the case when the damping is homogeneously distributed in the fiber and is characterized by a single parameter $\zeta(f)$.

We can then use Eqs. (11), (12) and (13) to compute the dissipated power, W_{diss} , from Eq. (9). To minimize the readout thermal noise $S_x(f, \alpha)$, we must choose the optimal $\alpha = \alpha_{\text{opt}}$ which minimizes W_{diss} , i.e. $\partial W_{\text{diss}} / \partial \alpha = 0$.

⁷We can justify this procedure by decomposing the motion of suspension fiber into normal modes, and considering how each mode is driven by the generalized force. When the driving frequency is close to a proper frequency of some violin mode, we can neglect excitation of other modes. Complexifying the frequency in the way described above is the standard procedure for finding the response of a damped harmonic oscillator close to its resonance frequency. Here by “close” we mean within a few widths $\gamma_n = \omega_n/Q_n$ of the n -th violin resonance; in our numerical evaluations we used the complex replacement within $10\gamma_n$ from n -th resonance.

IV. THE NEW READOUT VARIABLE: RESULTS

When we carry through the calculations outlined in the previous section, we find that the optimal readout variable which minimizes the thermal noise is

$$x_{\text{opt}} = x_{\text{mirror}} + \alpha_{\text{opt}} x_{\text{fiber}}, \quad (16)$$

where

$$\alpha_{\text{opt}} = A_{11} / (B_{11} + B_{12} + B_{13}); \quad (17)$$

$$A_{11} = -2gk \left\{ gk \cos[k(l - z_0)] + \omega^2 [\cos(kl) \sin(kz_0) - \sin[k(l - z_0)]] \right\}$$

$$B_{11} = g^2 k^2 \cos^2(kl) + 2g^2 k^2 \cos^2[k(l - z_0)]$$

$$B_{12} = \omega^2 [\omega^2 \sin^2(kl) - gk \sin(2kl) + 2\omega^2 \sin^2[k(l - z_0)]] \quad (18)$$

$$B_{13} = 2\omega^2 [\omega^2 \sin^2(kz_0) - gk \sin[2k(l - z_0)]] .$$

The optimal value α_{opt} is plotted in Fig. 3 for the case when the sensor is monitoring the midpoint of the fiber, i.e. $z_0 = l/2$. As is clear from the figure, α_{opt} is frequency-dependent, and one should keep this in mind while designing the numerical procedure for the data analysis. On resonance (where our method is expected to be most efficient), we get

$$\alpha_{\text{opt}} \simeq \frac{m}{\pi M} = 6.36 \times 10^{-6} \left(\frac{m}{0.3\text{g}} \right) \left(\frac{15\text{kg}}{M} \right), \quad (19)$$

which is in exact agreement with our more intuitive calculation in Eq. (5).

The thermal noise with and without the optimal monitoring, $S_x(f, \alpha_{\text{opt}})$ and $S_x(f, \alpha = 0)$ are plotted in Fig. 2 (solid and dashed lines respectively). We see that the optimal monitoring removes completely the first and other “odd” violin spikes from the suspension thermal noise spectrum. “Even” violin spikes can also be removed if the sensor is positioned off-center of the suspension fiber.

How precisely do we need to choose α , so that significant noise reduction is achieved? The spectral density $S_x(f, \alpha)$ is a quadratic function of α . We can use the fact that it has a minimum at $\alpha = \alpha_{\text{opt}}$ to write

$$S_x(f, \alpha) = S_x(f, \alpha_{\text{opt}}) + \Delta S(f) \left(\frac{\alpha - \alpha_{\text{opt}}}{\alpha_{\text{opt}}} \right)^2, \quad (20)$$

where $\Delta S(f) = S_x(f, \alpha = 0) - S_x(f, \alpha_{\text{opt}})$. When the noise reduction is effective, $\Delta S(f) \simeq S_x(f, \alpha = 0)$. From Eq. (20) we see that so long as

$$\left| \frac{\alpha - \alpha_{\text{opt}}}{\alpha_{\text{opt}}} \right| < \sqrt{\frac{S_x(f, \alpha_{\text{opt}})}{S_x(f, \alpha = 0)}}, \quad (21)$$

the thermal noise reduction will not be seriously compromised. If the above condition is not satisfied, then the thermal noise is reduced by a factor $\sim (\alpha - \alpha_{\text{opt}})^2 / \alpha_{\text{opt}}^2$ which depends only on how well the observer is able to tune α .

V. INFLUENCE OF THE SENSOR NOISE

So far we have assumed that the sensor, which is independently monitoring the fiber displacement, has an infinite precision, i.e. we have assumed that it does not introduce any noise of its own into the readout variable. In real life no sensor is perfect. The sensor readout is, in general, given by

$$x_{\text{sensor}} = x_{\text{fiber}} + n_s, \quad (22)$$

where n_s is the random noise introduced by the sensor. We assume that n_s is a Gaussian random variable with noise spectral density, $N_s(f)$.

The total readout variable is then

$$x_{\text{readout}} = x_{\text{mirror}} + \alpha(x_{\text{fiber}} + n_s), \quad (23)$$

and the total readout noise is then

$$S_{\text{total}}(f, \alpha) = S_x(f, \alpha) + \alpha^2 N_s(f). \quad (24)$$

A new optimal value α_{new} of α has to be evaluated, so that $\partial S_{\text{total}}(f, \alpha)/\partial \alpha = 0$. Using Eqs. (20) and (24), we get after straightforward algebra:

$$\alpha_{\text{new}}(f) = \frac{\Delta S(f)}{\Delta S(f) + \alpha_{\text{opt}}^2(f) N_s(f)} \alpha_{\text{opt}}(f). \quad (25)$$

The optimized noise in the total readout variable is then

$$S_{\text{total}}(f, \alpha_{\text{new}}) = S_x(f, \alpha_{\text{opt}}) + \text{Harm}\{\Delta S(f), N_s(f) \alpha_{\text{opt}}^2(f)\}, \quad (26)$$

where $\text{Harm}\{A, B\} = AB/(A + B)$ is a harmonic mean of A and B . We use the above equation to compute the solid curve in Fig. 2.

In the neighborhood of the violin resonance, $\Delta S(f) \simeq S_x(f, \alpha = 0)$. The criterion for achieving significant noise reduction is then

$$N_s(f) \ll S_x(f, \alpha = 0)/\alpha_{\text{opt}}^2(f). \quad (27)$$

On resonance [17],

$$S_x(f_v, \alpha = 0) = \frac{4k_B T Q m}{M^2 \omega_v^3 n^2 \pi^2} \quad (28)$$

and $\alpha_{\text{opt}} = (1/\pi)m/M$ (here n is the order of the violin mode). Then a first requirement for the sensor sensitivity⁸ (that is, the sensitivity for which significant noise reduction can be achieved on resonance) in Eq. (27) becomes

⁸Essentially, this requirement amounts to what one would expect intuitively: the sensor must be able to resolve the thermal motion of the fiber.

$$\sqrt{N_s(f_v)} \ll \frac{1}{n\pi} \sqrt{\frac{k_B T Q}{2\pi m f_v^3}} \simeq 4.2 \times 10^{-7} \text{cm}/\sqrt{\text{Hz}} \left(\frac{0.3\text{g}}{m}\right)^{1/2} \left(\frac{Q}{10^6}\right)^{1/2} \left(\frac{500\text{Hz}}{f_v}\right)^{3/2} \left(\frac{T}{300\text{K}}\right)^{1/2}. \quad (29)$$

Most sensors are broad-band, and their sensitivity is essentially f -independent for the range of detectable gravitational-wave frequencies. The precision of monitoring specified by Eq. (29) is achievable by currently available shadow sensors, which can measure with precision as high as $10^{-9}\text{cm}/\sqrt{\text{Hz}}$; therefore violin spikes can be reduced somewhat even using current technology.

However, we would ideally like to reduce the thermal violin spike below the shot noise level. Shot noise will dominate over all other noise sources except its violin spikes, at the violin-spike frequencies. When one represents it by a random displacement of an individual test mass, this shot noise for LIGO-II interferometers is [18]

$$\sqrt{S_{\text{shotnoise}}(f)} \simeq 1 \times 10^{-18} \text{cm}/\sqrt{\text{Hz}} \times \left(\frac{f}{500\text{Hz}}\right). \quad (30)$$

The sensor sensitivity requirement in Eq. (27) then becomes

$$\sqrt{N_s(f_v)} < \sqrt{S_{\text{shotnoise}}(f_v)}/\alpha_{\text{opt}} \simeq 2 \times 10^{-13} \text{cm}/\sqrt{\text{Hz}}. \quad (31)$$

Ageev, Bilenko, and Braginsky [16] have recently built an interferometer which can measure a fiber displacement with precision of $\sim 10^{-11}\text{cm}/\sqrt{\text{Hz}}$. They are hoping to improve this sensitivity by two orders of magnitude in the near future [19]. This would, in principle, make an interferometric sensor viable for our scheme.

However, in a narrow-band search the sensor-noise requirement is more severe than Eq. (31) by a factor ~ 5 , because the shot-noise spectral density is lower.

VI. DISCUSSION AND CONCLUSIONS

Among possible sources of periodic gravitational waves, the most promising are neutron stars with spin periods of 1 – 5 milliseconds [20]⁹. The frequencies of these stars' gravitational waves lie in the range which is pierced by the thermal violin spikes. Detection of these gravitational waves by LIGO will require changing the reflectivity and the location of the signal recycling mirror of the interferometer in such a way that the shot noise is reduced significantly in a narrow band around the gravitational-wave frequency. If a violin spike happens to be nearby, eliminating it might help the gravitational-wave detection and measurement. Our method allows one to completely eliminate violin spikes from the thermal noise spectrum, at least in principle. Figure. 2 illustrates this by showing a suspension thermal noise curve with and without compensation.

⁹This statement is based on current astronomical observations which use electromagnetic waves (light, radio waves, x-rays, etc.). Naturally, LIGO might change our notion about what the most promising gravitational-wave sources are.

In practice, though, we seek a non-trivial experimental design in which each suspension fiber is monitored at a single point by an independent sensor. High sensor sensitivity is the key for the method to be effective. For LIGO-II we need the noise spectral density introduced by each sensor to be less than $\sim 2 \times 10^{-13} \text{cm}/\sqrt{\text{Hz}}$; this would allow us to reduce the violin spike below the broad-band shot-noise sensitivity curve. In order to “bring” the violin spike below the *narrow-band* shot-noise sensitivity curve, we need even higher sensor sensitivity, by a factor of ~ 5 . Two types of sensors are currently being used for detection of the fiber displacement: the shadow sensors and the interferometric displacement sensors. The shadow sensors of $\sim 10^{-9} \text{cm}/\sqrt{\text{Hz}}$ have been demonstrated, and $10^{-10} \text{cm}/\sqrt{\text{Hz}}$ should be possible [21]; but this still is a far call from what we need for effective thermal noise compensation. Interferometric sensors, such as those developed at Moscow State University, look more promising. Ageev, Bilenko, and Braginsky [16] have achieved a sensitivity of $\sim 10^{-11} \text{cm}/\sqrt{\text{Hz}}$ for a sensor based on reflecting light from a polished spot on a steel wire. The current experiment in Moscow [19] hopes to achieve significantly higher sensitivity by attaching a tiny mirror at the center of the fiber¹⁰. It remains to be seen whether future interferometric sensor designs will allow our method to be practical for LIGO.

VII. ACKNOWLEDGEMENT

We thank Kip Thorne for helpful advice, and for comments on this manuscript. DHS gratefully acknowledges Michael Cross for useful discussions and support while this paper was being written. Vladimir Braginsky and David Shoemaker have provided us with useful information about interferometric and shadow sensors, respectively. YL thanks ITP at UC Santa Barbara for hospitality during his extended visit. This work is supported by NSF Physics grant, PHY-9900997, YL also has been supported by the Theoretical Astrophysics Center at UC Berkeley, and by NSF physics grant PHY-9907949 at ITP.

APPENDIX A: MOTION OF THE PERIODICALLY DRIVEN SUSPENSION FIBER

In this appendix we derive the expressions for θ_{top} , θ_{bottom} and θ_{middle} , Eqs. (11), (12), and (13) of the main text. These are the amplitudes of the oscillating top, bottom and middle bending angles respectively (see Fig. 1), when a periodic force $F = F_0 \cos(\omega t)$ is applied to the mirror, and simultaneously a force αF is applied to the point $z = z_0$ on the fiber at which the displacement sensor makes its measurement. In our derivation we follow the spirit of Appendix A in BLV.

For convenience, we complexify the driving force:

$$F = F_0 e^{i\omega t}. \quad (\text{A1})$$

¹⁰The mirror might introduce extra mass and mechanical friction to the center of the fiber, which would increase thermal noise in our readout variable. Techniques developed in this paper can be readily used to analyse theoretically thermal noise for such experimental set-up.

The equation of motion of the fiber with a force $\alpha F_0 e^{i\omega t}$ applied at $z = z_0$ is

$$\frac{\partial^2 x_f}{\partial t^2} = c^2 \frac{\partial^2 x_f}{\partial z^2} + \frac{\alpha F_0}{\rho} \delta(z - z_0) e^{i\omega t}, \quad (\text{A2})$$

where $x_f(z, t)$ is the horizontal displacement of the fiber at point z and time t , c is the velocity of a wave in the fiber, $c = \sqrt{glM/m}$, M is the mass of the test mass, m is the mass of the fiber, l is the length of the fiber, g is the earth's gravity, and ρ is the fiber's mass per unit length.

We look for a solution of Eq. (A2) in the form $x_f(z, t) = u(z)e^{i\omega t}$. Then for $0 < z < z_0$ we have

$$u(z) = A \sin(kz), \quad (\text{A3})$$

and for $z_0 < z < l$ we have

$$u(z) = B \sin(kz) + C \cos(kz), \quad (\text{A4})$$

where A , B , and C are constants to be determined, and $k = \omega/c$ is the wavenumber of the standing wave induced in the fiber.. The jump conditions at $z = z_0$ are:

$$u(z_0 + \epsilon) = u(z_0 - \epsilon), \quad (\text{A5})$$

$$\left. \frac{du}{dy} \right|_{z_0+\epsilon} - \left. \frac{du}{dy} \right|_{z_0-\epsilon} = \frac{\alpha F_0}{gM}. \quad (\text{A6})$$

Applying these jump conditions to Eqs (A3) and (A4), we get

$$A = B + C \cot(kz_0), C = \frac{\alpha F_0}{Mgk} \sin(kz_0). \quad (\text{A7})$$

To close this system of equations, we use Newton's second law for the test mass, projected onto the horizontal axis [cf. Eq. (35) of BLV]:

$$-\omega^2 u(l) + g \left(\frac{\partial u(z)}{\partial z} \right)_{z=l} = \frac{F_0}{M}. \quad (\text{A8})$$

By solving together Eqs. (A7) and (A8), we get

$$A = \frac{F_0}{M} \left[\frac{1 + \alpha \cos[k(l - z_0)] - (\alpha/gk) \omega^2 \sin[k(l - z_0)]}{gk \cos(kl) - \omega^2 \sin(kl)} \right], \quad (\text{A9})$$

$$B = \frac{F_0}{M} \frac{1 + \alpha [\sin(kl) \sin(kz_0) + (\omega^2/gk) \sin(kz_0) \cos(kl)]}{gk \cos(kl) - \omega^2 \sin(kl)}, \quad (\text{A10})$$

$$C = \frac{F_0 \alpha}{Mgk} \sin(kz_0) \quad (\text{A11})$$

Using Eqs. (A3), (A4), and (A9)-(A11), we can now work out the fiber's three angles of bent;

$$\theta_{top} = \left. \frac{\partial x}{\partial z} \right|_{z=0} = \frac{F_0}{M} \left[\frac{k [1 + \alpha \cos [k (l - z_0)]] - (\alpha/g) \omega^2 \sin [k (l - z_0)]}{gk \cos (kl) - \omega^2 \sin (kl)} \right], \quad (\text{A12})$$

$$\theta_{bottom} = \left. \frac{\partial x}{\partial z} \right|_{z=l} = \frac{F_0}{M} \left[\frac{k \cos (kl) + (\alpha/g) \omega^2 \sin (kl)}{kg \cos (kl) - \omega^2 \sin (kl)} \right], \quad (\text{A13})$$

and

$$\theta_{middle} = \left. \frac{\partial x}{\partial z} \right|_{z=z_0-\epsilon} - \left. \frac{\partial x}{\partial z} \right|_{z=z_0+\epsilon} = \frac{\alpha F_0}{gM} \quad (\text{A14})$$

These are the same as Eqs. (11), (12), and (13) of the main text.

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FIGURES

FIG. 1. Motion of the suspension and the test mass under the action of the generalized force conjugate to the readout variable $x_{\text{readout}} = x_{\text{testmass}} + \alpha x_{\text{fiber}}$.

FIG. 2. Suspension thermal noise with and without compensation. The dotted line is the usual uncompensated suspension thermal noise. The dashed line is for optimal compensation and a sensor that is infinitely precise. The solid line is for thermal noise compensation that is partially impeded by the sensor noise. For the case in the figure, the sensor is positioned in the middle of the suspension fiber; therefore, only violin modes with an even number of nodes get affected by the compensation. By shifting the sensor away from the fiber's middle one could compensate thermal noise in the “odd” violin peaks as well as the even ones.

FIG. 3. Optimized value of α , for the case when the sensor is perfect (dashed line) and when the sensor is intrinsically noisy (solid line); cf. Eq. (25).





